Cognitive radios with multiple antennas exploiting spatial opportunities

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Abstract

In this paper, the achievable rates of the so called “multiple-input multiple-output interference channel”, exploited by a couple of single antenna primary terminals and two antenna cognitive radios under specific interference constraints, are analyzed. In particular, by assuming perfect channel state information at the cognitive terminals, a closed form expression for a linear precoding and linear reception scheme, which guarantees to meet the achievable rates and no mutual interference between primary and cognitive terminals, is obtained. Numerical results regarding the effects of different fading channels and of an imperfect knowledge of the channel are provided to evaluate the performances of the proposed scheme in real environments.

keywords: multiple-input multiple-output (MIMO); cognitive radio (CR); spatial opportunity; MIMO interference channel (MIMO IC).

1 Introduction

In the past decade there has been an explosive growth in spectrum demand due to the deployment of a wide variety of wireless services. In order to guarantee coexistence, different services are allocated to different licensed frequency bands, leading to a more and more congested radio spectrum [1]. However, recent studies carried out by the Federal Communications Commission (FCC) indicate that many licensed frequency bands are underutilized [2]. This motivates the introduction of Cognitive Radio (CR) networks [3] which support opportunistic spectrum sharing [1, 4] by allowing secondary (unlicensed) users to access already licensed bands allocated to primary (licensed) ones [1, 2, 4].

The goal in opportunistic spectrum sharing is to enhance the utilization of the radio spectrum, exploiting the unused resources, without causing harmful interference to existing primary user [5], while guaranteeing an acceptable level of service to the secondary (cognitive) users [4]. Therefore, the detection of opportunities, i.e. licensed but unused resources [5], represents a key issue in this context. In practice, unfortunately, despite their potential impact on the practical development of CRs, high-performance algorithms which can be applied to any opportunity detection problem are still unavailable, since the existing ones are highly standard dependent [5–8]. Most of the prior research works on opportunistic spectrum sharing consider a single antenna at the primary and secondary transceivers and show that opportunities can be obtained, for example, in the frequency, time or code domains (or in a combination of them), depending on the primary user’s transmission [1].

The introduction of multiple antennas in complex environments, such as the ones where the CR systems are usually exploited, has been shown to lead to significant benefits [9,10]. Although

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MIMO techniques for CR applications has gained attention both from a theoretical and a practical perspective [1, 10] to provide a CR network with more degrees of freedom (DOFs) [10] in space, in addition to time, frequency and code domains [1], a few solutions [1, 4, 9–15] have been proposed and some problems still remain open [1, 11].

In some recent works, in order to evaluate the capacity of the secondary system in opportunistic spectrum sharing, some simplifying hypothesis are assumed. As an example, in [9–12], the capacity achieved by a CR system equipped with multiple antennas in a MIMO interference channel (MIMO IC) [10] is shown under the hypothesis that the message sent by the primary transmitter is known at the secondary transmitter. Although the indicated theoretical analyses point out the possibility of obtaining satisfactory improvements with respect to traditional solutions, these approaches suffer in practical CR scenarios where the required knowledge could be unavailable (for example, due to the lack of cooperation from primary terminals) and, as a consequence, effective opportunistic spectrum sharing could not be guaranteed [11].

In the study presented in [13], it is assumed that transmitters (primary or secondary) are unaware of the other transmitter’s data. Although it is shown that an interference free communication can be obtained in a MIMO X channel [13] by employing properly designed linear filters at both primary and secondary transceivers, this approach requires some additional processing at the primary (i.e. legacy) system which is not a desirable requirement in practical opportunistic spectrum sharing scenarios.

Other approaches, presented in [1, 4, 14, 15], do not require any cooperation from the legacy terminals and allow the coexistence of primary and secondary systems by implementing transmit beamforming or precoding [3], an interference cancellation technique for CR networks, exhaustively treated in [3], which properly set weights on the transmit antenna elements to steer the power in the directions of interest, minimizing the interference to the primary users and maximizing the received power at the secondary ones. In [1, 14], by assuming that the secondary receiver is not affected by the interference caused by the primary transmitter (such a channel is also known as MIMO Z channel [16]), an optimal transmission scheme at the secondary transmitter is designed to maximize the achievable rates at the secondary system and to meet a given interference constraint for the primary system [1, 14]. Unfortunately, in real CR scenarios primary transmission could affect, even heavily, the secondary receiver, resulting in harmful interference which could cause a fall of the capacity of the cognitive link. A more realistic spectrum sharing scenario is assumed in [4, 15] where the secondary receiver can be affected by the primary transmissions. In particular in [4], a multiantenna secondary transmitter is employed for multicast transmission to a set of secondary single antenna receivers. To this end, the design of the transmitter is formulated as an optimization problem in which the available power is steered toward the secondary receivers, while the sidelobes causing interference to primary receivers are limited. A complete theoretical and practical analysis of different formulations of the multicast beamforming problem (e.g. maintain the interference at the primary receivers below a given threshold, guarantee a certain level of service at the secondary system), is presented [4]. However, the considered optimization problems are non-convex and NP-hard and can be solved using semi-definite programming relaxation [4] only in an approximate way. Although the effectiveness of the approaches in [4, 15] is shown, and the interference caused from primary transmissions at the secondary receivers is considered as additive noise [4, 15] in the optimization process, single antenna secondary receivers are considered and therefore receive beamforming cannot be implemented to separate this interference from the signal of interest [3].

In this work cognitive transceivers are provided with multiple antennas allowing to exploit both transmit and receive beamforming. It is shown that, assuming perfect channel state information (CSI) (as usually considered in this context [1, 4, 10, 11, 13–15] to provide an upper bound for the achievable rates), the interference to primary and secondary receivers can be completely canceled by applying properly designed precoding and postcoding techniques at the CR system. In practice, it allows to transmit orthogonally with respect to the primary one providing a general opportunity detection phase (i.e. standard independent). In particular, cognitive terminals equipped with two antennas are considered because of the fundamental importance in practice, as a MIMO terminal with a low number of antennas is generally preferable for cost, complexity, and space reasons.
Moreover, since a terminal with two antennas can be realized by using a single dual-polarization antenna, the obtained results are of interest also for single-antenna terminals.

It is shown that orthogonal transmissions can be effectively obtained in a MIMO IC [10] without the explicit knowledge of the message transmitted by the primary user. The achievable rates of the proposed scheme can be obtained by enforcing no mutual interference between primary and cognitive terminals. In order to provide a solution of interest in real environments, the primary terminals are assumed to be, in the presence of secondary terminals, exactly the same as in the case of absence of the CR system.

For the first time to the best of authors’ knowledge, assuming perfect CSI at the two antennas cognitive terminals (i.e. knowledge of the channel between CR transmitter and receiver, primary transmitter and CR receiver, and CR transmitter and primary receiver), an exact closed form expression for a linear precoding and linear reception scheme to meet the achievable rates is obtained through a technique similar to that used in zero-forcing MIMO systems [1, 15, 17]. A set of numerical results on the achievable rates and on the effects of imperfect CSI on the proposed scheme are shown. Finally, strategies for the CSI estimation in the absence of any cooperation from primary terminals are discussed.

1.1 Notation

In the following $E\{\cdot\}$ is the expectation operator, vectors are represented in bold, $\cdot^T$ stands for transpose, $\cdot^\dagger$ for transpose and complex conjugate, $\|\cdot\|_2$ for 2-norm, $I_n$ is an $n \times n$ identity matrix, $|\cdot|$ is the determinant, and $\rho(\cdot)$ is the rank of a matrix.

2 Proposed architecture and related achievable rates

In the proposed scenario, shown in Fig. 1, a couple of CR terminals, equipped with two antennas, have to communicate, doing an additional processing that allows the CR transmitter to not interfere with the primary receiver and, at the same time, avoids interferences on the cognitive receiver from the primary transmitter: in the following we will refer to these specific interference constraints with the term coexistence conditions.

In the following the primary terminals are equipped with single antennas: multi-antenna primary terminals are not considered for simplicity, but the obtained results can be easily extended to this case.

![Figure 1: The considered MIMO channel model. Primary users are provided with a single antenna, while secondary (cognitive) users with two antennas.](image)

The reference scheme of Fig. 1 can be identified with a MIMO IC [10], characterized by the following input-output relationships:

\[ y_p = h_{p}^T x_c + h x_p + n_p \]  

\[ y_c = h x_p + H x_c + n_c \]
where $y_p \in \mathbb{C}$ and $x_p \in \mathbb{C}$ are respectively the received and transmitted complex baseband signals of the primary terminals, $y_c \in \mathbb{C}^2$ and $x_c \in \mathbb{C}^2$ the received and transmitted complex baseband signal vectors of the cognitive terminals, $H$ is the $2 \times 2$ complex matrix describing the channel between the cognitive terminals with entries $h_{jj}, h \in \mathbb{C}$ the complex coefficient describing the channel between the primary terminals, $h = (h_{11}, h_{22})^T$ and $h_\tau = (h_{\tau 1}, h_{\tau 2})^T$ are the complex vectors describing respectively the channel between the primary transmitter and the cognitive receiver and the channel between the cognitive transmitter and the primary receiver. In the following, $H, h_r$ and $h_c$ are considered constant (during symbol time) and known for the cognitive terminals, if not stated otherwise. Although this channel model is narrowband (all coefficients are frequency independent), it could be easily extended to multi-carrier systems by applying it on a sub-carrier basis [18]. $n_p \in \mathbb{C}$ and $n_c \in \mathbb{C}^2$ are the zero-mean complex Gaussian noise quantities [19] respectively for the primary and the cognitive receivers and $E\{n_p n_p^*\} = \sigma^2_p$ and $E\{n_c n_c^*\} = \sigma^2_c I_2$ will be assumed.

The objective of the present Section is to evaluate the achievable rates of the channel defined in (1) and (2), under the coexistence conditions defined above, and to develop a closed form expression for a linear precoding and postcoding at the secondary system which guarantees the achievable rates.

By using a technique similar to that used in zero-forcing based MIMO schemes [17], two “decoupling” $2 \times 2$ complex matrices $A$ and $B$ are introduced such that $x_c = Ax_a$ and $y_b = By_c$. Thus (1) and (2) become

$$y_p = h^T x_c + h x_p + n_p = h^T r x_a + h x_p + n_p$$  \hspace{1cm} (3)  
$$y_b = B y_c = B h x_p + BHA x_a + B n_c$$  \hspace{1cm} (4)  

and coexistence conditions are equivalent to (respectively)

$$\begin{cases}  
  h^T A = 0 \\
  B h_r = 0
\end{cases}$$  \hspace{1cm} (5)  

where $||A||_2 = 1$ has to be enforced in order to avoid signal amplification at the transmitter.

By enforcing (5) and by assuming that $h_{t,i} \neq 0$ and $h_{r,i} \neq 0$, $i = 1, 2$ (otherwise, a partial spatial orthogonalization is already performed by the channel), $A$ and $B$ can be expressed as,

$$A = \begin{bmatrix}
  a_{11} & -a_{22}h_{r,2} \\
  -a_{12}h_{r,1} & a_{22}
\end{bmatrix}, \quad B = \begin{bmatrix}
  b_{11} & -b_{22}h_{t,1} \\
  -b_{12}h_{t,2} & b_{22}
\end{bmatrix}$$  \hspace{1cm} (6)  

and, from (3) and (4) one obtains

$$y_p = h x_p + n_p$$  \hspace{1cm} (7)  
$$y_b = BHA x_a + B n_c$$  \hspace{1cm} (8)  

where (7) and (8) respectively represent a simple scalar AWGN channel for the primary terminals and a point-to-point MIMO channel defined by $\tilde{H} = BHA$ (with two inputs and outputs) for the CRs, now mutually decoupled. It is important to remark that, under the assumptions indicated above, the proposed linear technique allows us to separate the transmissions of primary and cognitive users with no hypothesis on their transmission standards and related messages, as required by the opportunistic spectrum sharing strategy.

To investigate the achievable rates of (8), it is known that the degrees of freedom (DOFs) of such a $2 \times 2$ channel can be exploited by using the singular value decomposition (SVD) of the channel matrix $\tilde{H}$, obtaining up to 2 parallel Gaussian channels [18–20]. This common procedure allows us to write $\tilde{H} = UDV^\dagger$ (where $D$ is a diagonal matrix containing the singular values of $\tilde{H}$, while $U$ and $V$ are unitary matrices) and, by introducing $x = V^\dagger x_a$ and $y = U^\dagger y_b$, to obtain (from (8))

$$y = U^\dagger y_b = U^\dagger \tilde{H} V x + U^\dagger B n_c = D x + U^\dagger B n_c.$$  \hspace{1cm} (9)
The achievable rates of this linear processing scheme, shown in Fig. 2, can be calculated by finding the matrices $A$ and $B$, satisfying (6), and the statistics of $x$ that maximize the mutual information between $x$ and $y$ under the considered hypotheses.

As it is shown in the Appendix, under the assumption of perfect CSI at the cognitive users, which is the only information used to perform the spatial orthogonalization, the achievable rates of the considered MIMO cognitive link, for given matrices $A$, $B$, satisfying (6) and $||A||_2 = 1$, and for a given channel realization, can be expressed as

$$C = \frac{1}{2} E_{H,r,h_r, h_t} \left\{ \log \left( 1 + \frac{\psi}{\lambda} \right) \right\}$$

(10)

where $\psi = d_{11}^2 P$ and $\lambda$ is the first element of the covariance $K_n$ of the noise $n$. In the Appendix it is shown that $K_n = \sigma_n^2 U^H B B^H U$ and that $d_{11}$ is the only possible non-trivial diagonal entry of $D$ which can be analytically computed as

$$d_{11} = \left( |a_{11}|^2 |r_{1,1}|^2 + |a_{22}|^2 |r_{2,2}|^2 \right)^{\frac{1}{2}} \cdot \left( |b_{11}|^2 |t_{1,1}|^2 + |b_{22}|^2 |t_{2,2}|^2 \right)^{\frac{1}{2}}$$

$$\frac{|h_{11}|^2 |r_{1,1} h_{r,1} - h_{12} h_{r,1} h_{t,2} + h_{22} h_{r,1} h_{t,2}|}{|h_{r,1} h_{r,2} h_{r,1} h_{r,2}|}$$

(11)

Equation (10) represents the achievable rates of the channel of interest obtained under the stringent assumed co-existence conditions enforced to guarantee no interference to primary users, as required by the opportunistic approach, and under the assumption that only linear pre- and post-processing is permitted.

It is worth noting that (10) is expressed in an implicit form, since the matrices $A$ and $B$ are still undetermined. In order to complete the analysis, an explicit expression for $C$ has to be found, the expressions for the matrices $A$ and $B$ which guarantees the achievable rates have to be deduced, and consequently $d_{11}$ and $\lambda$ has to be taken into account.

To evaluate $\lambda$, the following lemma, whose proof is provided in the Appendix, is useful:

**Lemma 1** Let $A$, $B$ and $H$ in $\mathbb{C}^{2 \times 2}$, with $||A|| = ||B|| = 0$. Let $U D V^H$ be the SVD of $B H A$. If $\rho(B H A) = 1$, then $U^H B B^H U$ has at least the last column and the last row equal to zero.

From Lemma 1 it is possible to infer that $K_n$ is a diagonal matrix if $d_{11} \neq 0$ (otherwise it is the null matrix) and $\lambda$ is the only non-trivial eigenvalue of $K_n$, which is equal to the non-trivial eigenvalue of $B B^H$ multiplied by $\sigma_n^2$ (since $U$ is unitary). Thus

$$\lambda = \sigma_n^2 \left[ |b_{11}|^2 |t_{1,1}|^2 + |b_{22}|^2 |t_{2,2}|^2 \right] \frac{|h_{t,1}|^2 + |h_{t,2}|^2}{|h_{t,1}|^2 |h_{t,2}|^2}.$$  

(12)

By substituting (11) and (12) into (10), one can deduce that

$$C = \frac{1}{2} E_{H,r,h_r, h_t} \left\{ \log \left[ 1 + \frac{P |a_{11}|^2 |h_{r,1}|^2 + |a_{22}|^2 |h_{r,2}|^2}{\sigma_n^2 F(H, h_r, h_t)} \right] \right\}$$

(13)

where

$$F(H, h_r, h_t) = \frac{|h_{r,1} h_{r,2} h_{t,1} h_{t,2} h_{r,1} - h_{12} h_{r,1} h_{t,2} + h_{22} h_{r,1} h_{t,2}|}{|h_{r,1} h_{r,2} h_{r,1} h_{r,2}|}.$$  

(14)
It can be verified that, when $d_{11} \neq 0$, for a given $A$ that satisfies the first of (5), $C$ depends only on $||A||_2$ since $B$, from (13) has no influence on $C$. This significantly simplifies the problem: as a matter of fact, the choice of optimal $A$ and $B$ does not require any optimization phase, but just the selection of $A$ and $B$ according to (6), with $||A||_2 = 1$.

3 Practical remarks and numerical results with Rice and Rayleigh fading

As (13) is channel dependent, a numerical analysis can be useful to evaluate the performances of the proposed MIMO CR in practical scenarios and its robustness to imperfect CSI.

Firstly, the achievable rates of the proposed scheme are compared with those of an alternative approach, time division duplexing (TDD), in order to evaluate under which conditions (e.g. different utilization of the primary link, fading effects) the proposed MIMO CR leads to valuable performance benefits with respect to traditional solutions [21]. In fact, while time opportunity is clearly defined in the open literature [6], for spatial opportunity further investigations are needed [21]. Moreover, a comparison between the opportunity detection and the opportunity exploitation phases of these two approaches is provided, and practical remarks on their implementation complexity are discussed.

The achievable rate region of the proposed scheme is equal to

$$C_1 = C_{CR,1} + C_{PU,1},$$

where $C_{PU,1}$ is the achievable rate region of the primary link and $C_{CR,1}$ is the achievable rate region of the cognitive link expressed by (13). Since the spatial orthogonalization does not require any information on the activity of the primary users, (15) is independent of the fraction of time the primary link is unused. On the contrary when TDD is performed, the cognitive users exploit the full $2 \times 2$ MIMO channel when available and therefore the achievable rate region of the cognitive link $C_{CR,2}$ is equal to that of a $2 \times 2$ MIMO channel $C_{MIMO}$ multiplied by the fraction of time the primary link is unused ($0 \leq \Delta t_u \leq 1$). Thus the achievable rate region when TDD is exploited is equal to

$$C_2 = C_{CR,2} + C_{PU,2} = \Delta t_u C_{MIMO} + (1 - \Delta t_u)C_{SISO} \leq C_{MIMO},$$

where $C_{PU,2}$ is the achievable rate region of the primary link in this scenario, and $C_{SISO}$ is the achievable rate region of a single-input single-output (SISO) Gaussian channel.

Numerical simulations have been carried out with different MIMO Rice and Rayleigh channels [23, 24] to evaluate the achievable rates for the proposed scheme (15) and the TDD approach (16). Each channel entry is an independent complex Gaussian variable with mean $\sqrt{K/(K+1)}$ and variance $1/(K+1)$ [23], where $K$ is the Rice factor. In Fig. 3 the values of $\Delta t_u$ for which $C_{CR,1} \geq C_{CR,2}$ and $C_1 \geq C_2$ are reported for each $K \in (0, 20)$, for values of signal to noise ratio (SNR) equal to 10 or 20 dB at the cognitive and at the primary receivers.

Fig. 3 shows that TDD has better performances than spatial orthogonalization for high values of $\Delta t_u$, while the contrary holds true for smaller values of $\Delta t_u$ ($C_1 > C_2$), for any SNR. Moreover, for values of $\Delta t_u$ below a threshold (rising with the value of SNR), spatial orthogonalization is significantly better than TDD ($C_1 > C_2$ and $C_{CR,1} > C_{CR,2}$) for any $K$. Therefore, the proposed MIMO CR can allow higher data rates than TDD if $\Delta t_u$ is below a certain threshold which, for example, varies from 0.72 to about 0.27 (for $K$ varying from 0 to 20) for SNR= 10 dB. This is not at odds with intuition: in fact, if the primary link is almost unused (high values of $\Delta t_u$), using the full $2 \times 2$ channel during primary inactivity gives a higher overall rates than that which can be obtained by spatially decoupling the transmission from a (mostly silent) primary link.

In the previous simulations perfect CSI was assumed. In practice, while the estimation of $H$ is a typical MIMO problem [18], the evaluation of $h_r$ and $h_t$ (only done by the cognitive terminals) has
never been faced before and may not be performed with a straightforward application of classical CSI estimation algorithms [4]. For this reason an analysis of the practical algorithms that could be used in the considered scenarios is at present under investigation. In particular, while the primary sender’s pilot symbols (typically used in narrowband multi-level modulations [18]) could be used to evaluate $h_f$ (the “primary-to-cognitive” channel), the estimation of $h_r$ (the “cognitive-to-primary” channel) seems much more difficult, at least without the cooperation of the primary receiver (silent by definition).

This suggests that spatial diversity could not be exploited under the coexistence conditions defined at the beginning of Section 2 if, as in broadcast applications, silent terminals are present: such a problem is common to any opportunistic CR scenario [5].

However, in point-to-point wireless applications, each receiver is generally expected to acknowledge the message reception [18]. In this scenario, and assuming channel reciprocity, the estimation of $h_r$ can be performed during the transmission phases of the primary “receiver” [4]. Unfortunately, the estimation of $h_r$ will be based on older measures than those used for $H$ and $h_f$. Various models [17, 18] could be used to evaluate the effect of imperfect CSI on the proposed scheme. Since analytical solutions for the achievable rates with imperfect CSI are difficult to obtain, in the present study a preliminary sensitivity analysis of the proposed scheme to outdated CSI is performed. In the following, the cognitive terminals are assumed to share the same outdated set of information $\hat{H}, \hat{h}_f, \hat{h}_r$, whose correlation coefficients to the true channel matrices or vectors are, respectively, $J_0(2\pi f_d \tau_r), J_0(2\pi f_d \tau_c)$ and $J_0(2\pi f_d \tau_c)$ [17], where $J_0$ is the zeroth order Bessel function of the first kind, $f_d$ is the Doppler frequency and $\tau, \tau_c, \tau_r$ are respectively the delay introduced between the estimation and exploitation of the matrices.

In the reported simulations, the signal to interference plus noise ratio (SINR) has been evaluated assuming SNR equal to 10 or 20 dB at the cognitive and at the primary receivers. The results obtained for $f_d \tau = f_d \tau_c \in [0, 0.5]$ and $f_d \tau_c \in [0, 0.5]$ are reported in Fig. 4. As it can be seen, the SINR at the cognitive and at the primary receivers degrades as the delay increases, for all values of SNR. This is due to the progressive loss of orthogonality between the primary and cognitive links as the accuracy of the available information degrades. Hence, outdated CSI has a similar effect on the SINR at the cognitive and at the primary receivers. As an example, in the case SNR = 10 dB, this can lead to a reduction of the SINR bigger than 7 dB for $f_d \tau$ (or $f_d \tau_c$) bigger than 0.3.

The reported results show that imperfect CSI can significantly affect the performances of the proposed scheme: however, such kind of degradation is a common problem in zero-forcing MIMO systems [17]. Some techniques for its mitigation are proposed in the open literature [17]: however, the adaptation of these techniques to the proposed scheme is out of the scope of the present paper.

Figure 3: Values of $\Delta t_u$ for which $C_{CR,1} \gtrsim C_{CR,2}$ and $C_1 \gtrsim C_2$, for different values of the Rice factor $K$ and of SNR.
This work allows to improve the result presented in [4], where the optimal tradeoff among transmitted power, SNR at the secondary users and interference at the primary users has been found by solving the optimization problem in an approximate way. As a matter of fact, by using a technique similar to zero-forcing beamforming, we obtain an exact solution which allows to cancel the interference to the primary receiver while maintaining a fixed transmitted power and maximizing the achievable rates. Finally, the interference to the secondary receiver is also canceled, since it is equipped with multiple antennas.

It is important to note that given $H$, $h_t$, and $h_r$, (13) can be used to establish if a spatial opportunity (i.e. a sufficient achievable rate of the link) is available. We note that such an approach is not as standard dependent as other opportunity detection techniques, since it deals with CSI rather than with spectral or temporal activity models of the primary users [6, 8]. This observation could be useful when comparing the complexity of the proposed scheme to that of a TDD scheme. In fact, while TDD transmission and reception is trivial, the proposed scheme requires careful CSI estimations (to calculate $A$, $B$). On the other hand, however, opportunity detection can be extremely complex in a TDD scenario [6], while, as discussed, detecting a “spatial opportunity”, given the channel state information, simply requires a comparison of the expected achievable rate (13) with a threshold. In this sense, such scheme shifts the complexity from “white space” [6] detection to CSI estimation.

The last observation regards the communication model considered in the analysis. As a matter of fact, in interference channel models [9, 10] simplex communication are usually considered. It could be of interest, however, to evaluate the impact of duplex communications on the proposed processing scheme. In particular, if primary users use an half duplex communication, the proposed algorithm can still be applied, by considering the same processing scheme but with different channels involved in the orthogonalization process. On the contrary, if full duplex communications of primary users are considered, the proposed processing scheme cannot be applied anymore because of the lack of DOFs required for decoupling primary from secondary transmission. However, spatial orthogonalization can still be performed, but a higher number of antennas is required. A deeper evaluation of such a scheme will be the subject of a future work.

4 Conclusions

In this paper, the achievable rates of a MIMO IC exploited by single antenna primary terminals and two antenna secondary terminals have been obtained by enforcing specific coexistence conditions. In particular, it has been shown that secondary terminals can transmit “orthogonally”
with respect to primary users, i.e. without causing interference on primary users and without suffering interference from the primary transmitter. This is obtained by properly designing an explicit linear precoding and linear reception scheme which guarantees the achievable rates when perfect CSI is assumed. It is important to note that the developed scheme does not require any knowledge about the primary terminals, and does not require any modifications at the primary terminals, as it is usually required in practical CR scenarios. Numerical results regarding the effects of different Rice and Rayleigh fading channels and of imperfect CSI have been reported to evaluate the performances of the proposed scheme in a real environment. Practical considerations on the CSI estimation and on the “spatial opportunity” detection problems have been discussed.

A Appendix: Derivation of (10)

By introducing \( w = Dx \) and \( n = U^T Bn_c \), (9) can be transformed into the following two parallel Gaussian channels with colored noise

\[
y = w + n. \tag{17}
\]

The theory of parallel Gaussian channels with colored noise is well established in the literature [20]. However, since in our case the constraints on (17) differ from the classical case (the power constraint is not given in terms of \( w \), as an example), the theoretical derivation has to be modified accordingly.

The covariance of the input \( w \) is \( E \{ww^\dagger\} = DK_x D \), where \( K_x = E \{xx^\dagger\} \), while \( K_n = E \{nn^\dagger\} = \sigma_n^2 U^T B B^\dagger U \) is the covariance of the noise \( n \). From the analysis of the theoretical capacity of the MIMO IC [10], one can deduce that the number of DOFs of channels expressed by (1) and (2) is equal to 2 and since the number of DOFs of the primary link is 1, \( D \) will have at most one non trivial diagonal entry \( d_{11} \), corresponding to the number of DOFs of the cognitive link. Hence one can deduce that

\[
E \{ww^\dagger\} = \begin{pmatrix} d_{11}^{\frac{1}{2}} E \{|x_1|^2\} & 0 \\ 0 & 0 \end{pmatrix} \tag{18}
\]

where \( E \{|x_1|^2\} \) is first entry of \( K_x \), being \( x_1 \) the first component of \( x \). From (18), no signal power will be received on the last component of \( y \) for any choice of \( K_x \). Accordingly, the achievable rates of (17) have to be evaluated by taking only the non trivial component of \( x \) into account. Channel (17), can be thus modified as follows

\[
y = w + n \tag{19}
\]

where (19) is then a scalar equation, the variance of the input \( w \) is \( E \{ww^*\} = d_{11}^{\frac{1}{2}} E \{|x_1|^2\} = \psi \), the variance of the output \( y \) is \( E \{yy^*\} = \xi \), and the variance of the noise \( n \) is \( E \{nn^*\} = \lambda \).

By imposing that the power transmitted by the cognitive terminal is less or equal to \( P [20] \), the following constraint will be obtained

\[
E \{|x_1|^2\} \leq P. \tag{20}
\]

Under this constraint, the achievable rates of the cognitive link obtained by employing the proposed linear scheme can be determined by finding \( A \) and \( B \) satisfying (6) and the input variance \( E \{|x_1|^2\} \in \mathbb{R} \) such that the mutual information \( I(w; y) \) and \( y \) in (19) is maximum, subject to (20) and \( \| A \|_2 = 1 \).

The mutual information \( I \) of the cognitive link (19) is equal to

\[
I(w; y) = H_y(y) - H_n(n) \tag{21}
\]

where the input and the noise are assumed to be independent, \( H_y(y) \) and \( H_n(n) \) represent the entropy of the output and of the noise of channel (19), respectively. As a consequence [20], \( H_n(n) = \frac{1}{2} \log (2\pi e \lambda) \). Usually [20], \( H_n(n) \) is not considered in the maximization problem (since it is independent of the input distribution), but in the proposed case \( \lambda \) depends on \( A \) and \( B \) and has to be taken into account.
By following a technique similar to [20], which shows that the entropy of $y$ is maximized when $w$ is normal and thus when also $y$ is normal, one can show that $\mathcal{H}_y(y) \leq \frac{1}{2} \log (2\pi e \xi)$, where the equality holds if and only if the input is Gaussian.

Moreover, since the input and the noise are independent, $\xi = \lambda + \psi$; substituting $\mathcal{H}_n(n)$ and $\mathcal{H}_y(y)$ into (21), one can obtain

$$I(w; y) \leq \frac{1}{2} \log \left( \frac{\lambda + \psi}{\lambda} \right) = \frac{1}{2} \log \left( 1 + \frac{\psi}{\lambda} \right),$$

where the equality holds if and only if the input is Gaussian.

Moreover, (20) can be arranged in terms of $\psi$ as:

$$E \{ |x_1|^2 \} = \frac{\psi}{d_{11}} \leq P.$$  

From (22), by taking into account the statistical variations of the channel one deduces

$$C = E_{H, h, n} \left[ \max_{A, B, \psi} \frac{1}{2} \log \left( 1 + \frac{\psi}{\lambda} \right) \right]$$

under the constraints (23) and $||A||_2 = 1$.

For fixed $A$ and $B$, which, as remarked previously, have to satisfy (6) and the condition $||A||_2 = 1$, the solution to (24) is found as the optimal input variance $\psi$ for the parallel Gaussian channel subject to (23). Therefore, the achievable rates of the MIMO cognitive link with the proposed linear processing scheme and $d_{11} \neq 0$ can be expressed by

$$C = E_{H, h, n} \left[ \frac{1}{2} \log \left( 1 + \frac{\psi}{\lambda} \right) \right]$$

where all power has to be transmitted over $x_1$ (the first component of $x$) since $\psi = d_{11}^2 E \{ |x_1|^2 \} = d_{11}^2 P$ (from (20)) and $\lambda$ is the first element of $K_n = \sigma_n^2 U^* BB^* U$.

### B Appendix: Proof of Lemma 1

Let $\Gamma = U^* B = [\gamma_U | \gamma_L]^\dagger \in \mathbb{C}^{2 \times 2}$, where $\gamma_U \in \mathbb{C}^2$, $\gamma_L \in \mathbb{C}^2$, and let $\Theta = HA \in \mathbb{C}^{2 \times 2}$. Then

$$U^* BB^* U = \begin{pmatrix} \gamma_U^\dagger \gamma_U & \gamma_L^\dagger \gamma_L \\ \gamma_U^\dagger \gamma_L & \gamma_L^\dagger \gamma_L \end{pmatrix}. \tag{26}$$

Let $V = [v_s | v_d] \in \mathbb{C}^{2 \times 2}$, where $v_s \in \mathbb{C}^2$, $v_d \in \mathbb{C}^2$. By exploiting $\rho(BHA) = 1$ we deduce that $D$ has exactly one non trivial diagonal entry $d_{11}$. Then

$$DV^\dagger = \begin{pmatrix} d_{11} v_s^\dagger \\ 0^\dagger \end{pmatrix} \tag{27}$$

it is different from zero since $d_{11} \neq 0$ and $v_s \neq 0$ being $V$ unitary. From the hypotheses, and using (27), one can obtain

$$U^* BHA = \Gamma \Theta = \begin{pmatrix} \gamma_U^\dagger \\ \gamma_L^\dagger \end{pmatrix} \Theta = DV^\dagger = \begin{pmatrix} d_{11} v_s^\dagger \\ 0^\dagger \end{pmatrix} \tag{28}$$

which can be rewritten as follows

$$\begin{align*}
\Theta^\dagger \gamma_U &= d_{11} v_s^* \\
\Theta^\dagger \gamma_L &= 0.
\end{align*} \tag{29}$$

By contradiction suppose that $\gamma_L \neq 0$. Since $|\Gamma| = 0$, we have necessarily $\gamma_U = \alpha \gamma_L$, $\alpha \in \mathbb{C}$. If $\alpha = 0$ we deduce $\gamma_U = 0$ and the first of (29) is violated. On the other hand, if $\alpha \neq 0$, by using the first of (29), $0 \neq d_{11} v_s^* = \Theta^\dagger \gamma_U = \Theta^\dagger \alpha \gamma_L$ and this violates the second of (29). Therefore $\gamma_L = 0$ and, from (26), $U^* BB^* U$ has at least the last column and the last row equal to zero.
References


