Electromagnetic inverse scattering of axially moving cylindrical targets

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Abstract

Electromagnetic inverse scattering techniques are considered to reconstruct the permittivity and the velocity profiles of axially moving cylindrical targets. Two approaches are proposed. One of these is based on a two-step procedure. It is shown that it provides good approximations of the profiles to be reconstructed in a very efficient way, when the peak velocity is small with respect to the speed of light in vacuum. These features are obtained by neglecting the movement in the first step, which is devoted to the reconstruction of the geometric and dielectric properties of the cylindrical targets. The second approach is more classical and is defined as a global optimization problem. It is shown that the two procedures are in some way complementary in the sense that when the two-step procedure can be applied it is by far the best, with the general procedure giving large errors; vice versa, the two-step procedure is in trouble when the other one gives good results. In carrying out this analysis some estimates on the effects of the movement on the scattered field components are deduced in the limit as the peak velocity goes to zero.

Keywords: time-harmonic electromagnetic scattering; bianisotropic media; moving media; inverse scattering procedures; reconstruction of permittivity and velocity profiles.

1 Introduction

A very important topic in electromagnetics is related to the interaction of electromagnetic waves with moving bodies [1]. In most cases, such an interaction is managed by considering problems formulated in the time domain [1]. For some particular models, however, the movement takes place in such a way that it is still possible to work in the frequency domain [2], [1].

Among these models a particular interest have been devoted to those involving axially moving cylinders. As a matter of fact, the electromagnetic scattering from axially moving cylinders has important applications in astrophysics, nuclear and plasma physics, and engineering. In particular, some applications are related to the reconstruction of the dielectric and velocity profiles of ionized meteor trails [3], axially moving plasma columns [4], [5], [6], jet exhausts [7], mass flows in pneumatic pipes [8].

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In the last decades several techniques have been considered in order to determine the dielectric and velocity profiles [8] of axially moving cylinders. In particular, depending on the type of reconstruction of interest, one can use acoustic or electromagnetic waves. The latter type of waves present less limitations (in space and astrophysics applications, for example) and is preferred in most cases [8], [4], [5]. Different electromagnetic techniques have been exploited so far. One can refer to techniques based on signals in the microwave or optical bands [8] or to approaches based on the Doppler effects. Many other possibilities have been considered and can be found in the open literature [8].

Microwave inverse scattering techniques [9] have been proved to possess excellent properties in the reconstruction of permittivity profiles in breast cancer detection [10], [11], ground penetrating radar applications [12], [13], [14], [15], [16], [17], crack detection [18], and so on. It seems, however, that such techniques have not been used so far for the solution of inverse scattering problems involving axially moving cylinders, when the reconstruction of the velocity profile is of interest.

For this reason, in this work we will perform a study on the capabilities of electromagnetic inverse scattering techniques to deal with problems involving the reconstruction of velocity profiles. Unfortunately, the most general formulation of the problems of interest is still too complex: axially moving cylinders could be made up of anisotropic and inhomogeneous media, with very complex geometries and velocity profiles. These difficulties could hide the advantages and disadvantages of inverse scattering techniques with respect to other approaches, for the problems of interest. In order to try to obtain such a kind of result and to deduce the first indications on the performances which can be achieved by inverse scattering methods, we simplify the models of interest by assuming that all media involved are linear, stationary, non-dispersive and isotropic in their rest frames and that the permittivity and the velocity profiles are piecewise constant functions. With the same spirit, we will also assume that the geometry can be defined with a finite set of parameters.

With the indicated assumptions, the inverse scattering problems can be recast as a reconstruction problem with a finite number of unknowns. Some of these unknowns are related to the geometry or to the dielectric properties; some others are related to the velocity profiles. In this paper we will firstly analyze a general approach in which all unknowns are managed in the same way. It will be shown that such a technique could be able to deal with problems involving moving media at relativistic velocities. However, it will also be shown that this strategy can be in trouble when the axial movement takes place with velocities which are very small if compared to the velocity of light. In these cases another methodology will be shown to provide much better results. This alternative approach is based on a two-step approximate procedure: the first one determines the geometric and dielectric quantities of the reconstruction problem and the second one calculates the quantities related to the velocity profiles. In particular, the first inversion procedure considers all media at rest and exploit the measurements of just one axial component, when the illuminating field is transverse-magnetic or transverse-electric. The indicated measurements refer, however, to the problem of interest, when the scatterer is in motion. The measurements of the other axial component are then exploited by the second step.

In order to obtain the results of interest, in Section 2 we deduce some estimates on the solutions of forward scattering problems in the presence of axially moving cylindrical scatterers. These results do not require any assumption on the permittivity or velocity profiles or on the geometry of the scatterer. In Section 3 we present several numerical results, obtained in the presence of canonical scatterers with piecewise constant velocity profiles, to provide a first statistical evaluation of the effects of the axial movement on the field components. Two inverse scattering approaches are presented in Section 4. Finally, before the conclusions, in Section 5 we present the results related to the reconstruction of the permittivity and velocity profiles, in the presence of canonical and axially moving scatterers.
2 Considerations on the forward scattering problem

It is well known that for a generic linear, stationary, non-dispersive and isotropic medium characterized by \( \varepsilon_r, \mu_r, \sigma = 0 \) in its rest frame and moving with velocity given by \( \beta = \frac{v}{c_0} \) along the \( z \) axis the constitutive relation are given by \([19], [20], [5]\):

\[
D_t = \alpha \varepsilon_0 \varepsilon_r E_t + \frac{\delta}{c_0} \hat{z} \times H_t, \tag{1}
\]
\[
D_z = \varepsilon_0 \varepsilon_r E_z, \tag{2}
\]
\[
B_t = \alpha \mu_0 \mu_r H_t - \frac{\delta}{c_0} \hat{z} \times E_t, \tag{3}
\]
\[
B_z = \mu_0 \mu_r H_z, \tag{4}
\]

where \([20], [5]\):

\[
\alpha = \frac{1 - \beta^2}{1 - \mu_r \varepsilon_r \beta^2} \tag{5}
\]
\[
\delta = \beta \frac{(\mu_r \varepsilon_r - 1)}{1 - \mu_r \varepsilon_r \beta^2}. \tag{6}
\]

For a scattering problem involving materials having the indicated constitutive relations in a domain \( \Omega_s \) we can proceed as in \([21]\) (Section 7.7, pp. 327-328) by defining the electromagnetic field \((E, H)\), the incident electromagnetic field \((E_i, H_i)\) and the scattered electromagnetic field \((E_s = E - E_i, H_s = H - H_i)\). Here we assume that these fields exist and are bounded in \( \Omega_s \).

For the cases of canonical scatterers this is proved in \([6]\) and \([22]\). To prevent the problem from being trivial we also assume that \( \mu_r \varepsilon_r \neq 1 \) somewhere in \( \Omega_s \).

By denoting with \( \varepsilon_{r,b} \) and \( \mu_{r,b} \) the relative permittivity and permeability of the homogeneous background medium one can easily deduce that the scattered electromagnetic field satisfies

\[
\nabla \times E^s = -j \omega \mu_0 \mu_r \varepsilon_r H^s - J_{m,eq}, \tag{7}
\]
\[
\nabla \times H^s = j \omega \varepsilon_0 \varepsilon_{r,b} E^s + J_{e,eq}, \tag{8}
\]

where

\[
J_{m,eq} = j \omega \mu_0 (\mu_r - \mu_{r,b}) H - j \omega \mu_0 (1 - \alpha) \mu_r H_t - j \frac{\omega}{c_0} \delta \hat{z} \times E_t, \tag{9}
\]
\[
J_{e,eq} = j \omega \varepsilon_0 (\varepsilon_r - \varepsilon_{r,b}) E - j \omega \varepsilon_0 (1 - \alpha) \varepsilon_r E_t + j \frac{\omega}{c_0} \delta \hat{z} \times H_t. \tag{10}
\]

In addition to the problem with the moving scatterer we can refer to the model in which the same scatterer is considered everywhere at rest and is illuminated by the same sources. For this model we define the electromagnetic field \((E_0, H_0)\) and the scattered electromagnetic field \((E_0^s = E_0 - E^s, H_0^s = H_0 - H^s)\). The incident field by definition is the same as before. For these fields, again from \([21]\), we deduce

\[
\nabla \times E_0^s = -j \omega \mu_0 \mu_r \varepsilon_r H_0^s - J_{m,eq,0}, \tag{11}
\]
\[
\nabla \times H_0^s = j \omega \varepsilon_0 \varepsilon_{r,b} E_0^s + J_{e,eq,0}, \tag{12}
\]

where

\[
J_{m,eq,0} = j \omega \mu_0 (\mu_r - \mu_{r,b}) H_0, \tag{13}
\]
\[
J_{e,eq,0} = j \omega \varepsilon_0 (\varepsilon_r - \varepsilon_{r,b}) E_0. \tag{14}
\]
From the previous two sets of equations, by observing that
\[ \mathbf{H} - \mathbf{H}_0 = \mathbf{H}^* - \mathbf{H}_0^* \] (15)
and
\[ \mathbf{E} - \mathbf{E}_0 = \mathbf{E}^* - \mathbf{E}_0^* \] (16)
one easily deduces
\[ \nabla \times (\mathbf{E}^* - \mathbf{E}_0^*) = -j \omega \mu_0 \mu_r (\mathbf{H}^* - \mathbf{H}_0^*) - \mathbf{J}_{m,eq,d}, \] (17)
\[ \nabla \times (\mathbf{H}^* - \mathbf{H}_0^*) = j \omega \varepsilon_0 \varepsilon_r (\mathbf{E}^* - \mathbf{E}_0^*) + \mathbf{J}_{e,eq,d}, \] (18)
where
\[ \mathbf{J}_{m,eq,d} = j \omega \mu_0 (1 - \alpha) \mu_r \mathbf{H}_t + j \frac{\omega}{c_0} \hat{z} \times \mathbf{E}_t, \] (19)
\[ \mathbf{J}_{e,eq,d} = -j \omega \varepsilon_0 (1 - \alpha) \varepsilon_r \mathbf{E}_t + j \frac{\omega}{c_0} \hat{z} \times \mathbf{H}_t. \] (20)

In particular, the two equivalent sources \( \mathbf{J}_{m,eq,d} \) and \( \mathbf{J}_{e,eq,d} \) have support in \( \mathbb{R}^2 \), have only the transverse components, depend just on the transverse coordinates and radiate in the presence of the scatterer at rest.

Since \( 1 - \alpha \propto \beta^2 \) and \( \delta \propto \beta \), at least for small values of \( \beta \) in \( \Omega_s \), thanks to the assumed boundedness of \( \mathbf{E} \) and \( \mathbf{H} \) in \( \Omega_s \) we obtain
\[ \lim_{\sup_{r \in \Omega_s} |\beta(r)| \to 0} || \mathbf{J}_{m,eq,d} ||_{0,\Omega_s} = 0, \] (21)
\[ \lim_{\sup_{r \in \Omega_s} |\beta(r)| \to 0} || \mathbf{J}_{e,eq,d} ||_{0,\Omega_s} = 0, \] (22)
where \( || \cdot ||_{0,\Omega_s} \) is the usual \( (L^2(\Omega_s))^3 \) norm. Then, due to the well posedness of the radiation problem in the presence of a traditional scatterer at rest, we conclude
\[ \lim_{\sup_{r \in \Omega_s} |\beta(r)| \to 0} || \mathbf{E}^* - \mathbf{E}_0^* ||_{\text{curl,loc}} = 0, \] (23)
\[ \lim_{\sup_{r \in \Omega_s} |\beta(r)| \to 0} || \mathbf{H}^* - \mathbf{H}_0^* ||_{\text{curl,loc}} = 0, \] (24)
where \( || \cdot ||_{\text{curl,loc}} \) is the norm of the space of vector fields \( H_{\text{loc}}(\text{curl}, \mathbb{R}^2) \) (that is the space of vector fields \( \mathbf{u} \) such that \( \mathbf{u}|_{\Omega} \in H(\text{curl}, \Omega) \) for all bounded and open \( \Omega \subset \mathbb{R}^2 \) [23] (p. 230)). Then, by (15) and (16)
\[ \lim_{\sup_{r \in \Omega_s} |\beta(r)| \to 0} || \mathbf{E} - \mathbf{E}_0 ||_{\text{curl,loc}} = 0, \] (25)
\[ \lim_{\sup_{r \in \Omega_s} |\beta(r)| \to 0} || \mathbf{H} - \mathbf{H}_0 ||_{\text{curl,loc}} = 0. \] (26)

Sharper estimates can be deduced, for example, if we consider incident fields having a TM (TE) polarization. As a matter of fact, for a non-trivial TM (TE) incident field we have \( H_{0,z} = 0 \) and \( E_{0,t} = 0 \) \( (E_{0,z} = 0 \) and \( H_{0,t} = 0 \) everywhere and, by (25) ((26)),
\[ \lim_{\sup_{r \in \Omega_s} |\beta(r)| \to 0} || \mathbf{E}_t ||_{\text{curl,}\Omega_s} = 0 \] (27)
\[ \left( \lim_{\sup_{r \in \Omega_s} |\beta(r)| \to 0} || \mathbf{H}_t ||_{\text{curl,}\Omega_s} = 0 \right). \] (28)
Since, on the contrary, \( \lim_{|\mathbf{r}| \to 0} \| \mathbf{H}_i \| \mathbf{curl} \Omega_i \neq 0 (\lim_{|\mathbf{r}| \to 0} \| \mathbf{E}_i \| \mathbf{curl} \Omega_i \neq 0) \),
as a consequence of (19), (20) and of the fact that \( 1 - \alpha \propto \beta^2 \), \( \delta \propto \beta \), for small values of \( \sup_{\mathbf{r} \in \Omega_i} |\beta(\mathbf{r})| \to 0 \), we conclude that \( \mathbf{J}_{m,eq,d} \) and \( \mathbf{J}_{e,eq,d} \) converge uniformly to zero more quickly than \( \mathbf{J}_{r,eq,d} \) when \( \sup_{\mathbf{r} \in \Omega_i} |\beta(\mathbf{r})| \to 0 \).

Now it is easy to verify [21] that \( \mathbf{J}_{m,eq,d} \) and \( \mathbf{J}_{e,eq,d} \) with the properties indicated below equation (20) is responsible for the excitation of the TM (TE) part of the wave \( (\mathbf{E}^* - \mathbf{E}^*_0, \mathbf{H}^* - \mathbf{H}^*_0) \). Thus, our previous considerations on the speed of convergence to zero of the indicated norms in the presence of a TM (TE) illuminating field allow us to conclude that, when \( \sup_{\mathbf{r} \in \Omega_i} |\beta(\mathbf{r})| \to 0 \), the effects of the axial movement on \( E_z - E_{0,z} = E_z^* - E_{0,z}^* \) \( (H_z - H_{0,z} = H_z^* - H_{0,z}^*) \) become negligible with respect to those on \( H_z = H_{0,z} = H_z^* = H_{0,z}^* \) \( (E_z = E_{0,z} = E_z^* - E_{0,z}^*) \).

### 3 Statistical evaluation of the effects of the axial speed values on the scattered field components for multilayer elliptic cylinders

Section 2 provides some indications on the properties of the scattered field in the limit as \( \sup_{\mathbf{r} \in \Omega_i} |\beta(\mathbf{r})| \to 0 \). Even though such indications are important, in order to fully exploit them in practical applications we need to define a “rule of thumb”, as simple as possible, allowing us to establish when a set of values of the axial velocity could be considered as a good approximation of the limit condition indicated above, so as to be able to reliably apply the theoretical deductions of Section 2. In this section we work towards the definition of such a criterion by computing the results of many numerical simulations. The set of scattering problems considered should be large enough to allow us to extrapolate a reliable rule of thumb for practical applications. In order to achieve this result we need to use a reliable but also fast forward scattering solver. For this reason our simulations will be performed by using the efficient recursive procedure described in [22], which assumes the presence of a \( N \)-layer elliptic cylinder. \( d \) denotes the semifocal distance of the confocal ellipses and \( a_i, i = 1, \ldots, N \), denotes the length of the semimajor axis of the interface between layer \( i \) and layer \( i + 1 \), the background medium being considered as layer number \( N + 1 \).

Each layer is considered to be homogeneous, characterized in its rest frame by \( \epsilon_{r,i} \in \mathbb{R}, \mu_{r,i} = 1 \) and \( \sigma_i = 0, i = 1, \ldots, N \), and to move in the axial direction with respect to the reference frame adopted with a uniform velocity \( \beta_i \in (-1, 1), i = 1, \ldots, N \). The external medium is assumed to be in any case the empty space.

We will carry out this analysis by focusing on a TM polarization for the incident field, since the effects for a TE polarization are dual, as shown in Section 2.

We hope to define a sufficiently large set of simulations by firstly considering \( N = 3 \), \( d = 0.15\lambda_0, a_1 = 0.2a_0, a_2 = 0.25\lambda_0, a_3 = 0.3\lambda_0 \), with \( \beta_i = 1, \ldots, 3 \), varying in the set \( \{0, 0.001, 0.002, 0.004, 0.008, 0.016, 0.032, 0.05, 0.08, 0.11, 0.14\} \) and \( \epsilon_{r,1} = \epsilon_{r,2} = \epsilon_{r,3} \) equal to 2 or 4. Then, in order to change the number of layers and the transverse dimensions of the scatterer we consider as the case in which \( N = 1 \), with \( d, a_1, \epsilon_{r,1} \) and \( \beta_i \) as before. Finally, the illuminating field could be a (TM) uniform plane wave \( (E_z^* = e^{j\frac{2\pi r}{\lambda_0}}) \) or the wave generated by an electric line current placed at a distance of \( 0.5\lambda_0 \) from the center of gravity of the scatterer, with an angular position with respect to the straight line passing through the focus of \( \frac{\pi}{2} \) and with a current flowing in the radiating element equal to \( \frac{1}{2\mu_0} [A] \). Overall, we consider \( 11^3 \cdot 2 \cdot 2 = 5324 \) cases when \( N = 3 \) and \( 11 \cdot 2 \cdot 2 = 44 \) cases when \( N = 1 \).

In the following all solutions have been computed by truncating all series expansions after the first 8 modes. It has been verified that this number is sufficient to give very precise solutions in all cases considered.
Figure 1: Behaviour of the root mean square values of $|E_z^s - E_{0,z}^s|$ and $|H_z^s|Z_0$ versus $\sup_{r \in \Omega} |\beta(r)|$. Each of the reported rms values has been determined by evaluating the indicated quantities on a set of 360 points uniformly distributed on a circle of radius 0.6$\lambda_0$, in the presence of a scatterer having three layers with $\varepsilon_{r,1} = \varepsilon_{r,2} = \varepsilon_{r,3} = 4$, or one layer with $\varepsilon_{r,1} = 4$. In all cases the scatterer is illuminated by an electric line source.

For all these problem configurations we evaluate, according to the deduction of Section 2, $|E_z^s - E_{0,z}^s|$ and $|H_z^s|Z_0$ near the scatterer, on a set of 360 points uniformly distributed on a circle of radius 0.6$\lambda_0$, and $\sqrt{r}|E_z^s - E_{0,z}^s|$ and $\sqrt{r}|H_z^s|Z_0$ far from it, on a set of 360 uniformly distributed points on a circle of radius $r$, for $r \to +\infty$. In the previous expressions the symbol $Z_0$ refers to $\sqrt{\mu_0 / \varepsilon_0}$. The different regions of evaluation are chosen for their importance for practical inverse scattering problems (astrophysics versus plasma physics, for example). Then, for any problem configuration, we evaluate the root mean square (rms) of the indicated sequences of 360 quantities evaluated in the near-field or in the far-field regions.

In Figure 1 and 2 we show the behaviour of the indicated rms values versus $\sup_{r \in \Omega} |\beta(r)|$. In particular, Figure 1 (respectively, 2) is obtained by considering just the quantities computed in the near-field (respectively, far-field) region. Moreover, such figures are obtained by considering the cases in which the scatterer has one or three layers, with $\varepsilon_{r,1} = 4$ when $N = 1$ and $\varepsilon_{r,1} = \varepsilon_{r,2} = \varepsilon_{r,3} = 4$ when $N = 3$, and is illuminated by the electric line current source defined as above. When $N = 1$ we have just one rms result for any $\sup_{r \in \Omega} |\beta(r)|$ value whereas, when $N = 3$, many rms values are shown for any value of the same quantity. As can be seen, there is not a significant difference between Figures 1 and 2. In particular, in both figures the quadratic behaviour of $|E_z^s - E_{0,z}^s|$ and $\sqrt{r}|E_z^s - E_{0,z}^s|$ and the linear one of $|H_z^s|Z_0$ and $\sqrt{r}|H_z^s|Z_0$ with respect to $\sup_{r \in \Omega} |\beta(r)|$ are clearly shown when $N = 1$ and on average when $N = 3$, at least for the smallest values of $\sup_{r \in \Omega} |\beta(r)|$ here considered. In order to clearly show such a statistical behaviour in Figure 3 we report the average values of the many results which are present in Figure 2 (essentially the same figure would come out by considering the data of Figure 1), for any $\sup_{r \in \Omega} |\beta(r)|$ value when $N = 3$. The results related to $N = 1$ are the same as those of Figure 2. The quadratic and linear effects are now clearer and the change of the number of
The conclusion we can draw from this statistical investigation is that the axial motion have negligible effects on $|E^s_z - E^s_{0,z}|$ or $\sqrt{\tau}|E^s_z - E^s_{0,z}|$ (respectively, on $|H^s_z - H^s_{0,z}|$ or $\sqrt{\tau}|H^s_z - H^s_{0,z}|$).
Figure 3: Average behaviour of the rms values of \( \sqrt{r}|E^s_z - E^s_0,z| \) and \( \sqrt{r}|H^s_z| Z_0 \) shown in Figure 2 when the scatterer has three layers and \( \varepsilon_{r,1} = \varepsilon_{r,2} = \varepsilon_{r,3} = 4 \). The graphs corresponding to \( N = 1 \) is the same as the one shown in Figure 2 and it is here reported to have a unique overview on the behaviour of the computed data.

Table 1: Largest normalized rms values versus \( \sup_{r_t \in \Omega_s} |\beta(r_t)| \). The largest rms values are taken from Figures 1 and 2. They are normalized, respectively, with the rms value of \( |E^s_0,z| \) or \( \sqrt{r}|E^s_0,z| \).
for TM (respectively, TE) incident fields provided that \( \sup_{r \in \Omega} |\beta(r)| \leq 0.01 \).

4 Different approaches for the reconstruction of the permittivity and velocity profiles

As pointed out in Section 1 the problem of interest concerns the reconstruction of a finite number of parameters related to the geometry, the dielectric properties and the velocity profile.

In particular, we assume that the unknowns of the inverse scattering problems of interest are given by the algebraic vector \( \mathbf{x} = (g_1, \ldots, g_T, \varepsilon_{r,1}, \ldots, \varepsilon_{r,J}, \beta_1, \ldots, \beta_K) \in \mathbb{R}^{I+J+K} \). This vector of unknowns can be split into its geometrical part \( (\mathbf{x}_g = (g_1, \ldots, g_T) \in \mathbb{R}^I) \), its dielectric part \( (\mathbf{x}_e = (\varepsilon_{r,1}, \ldots, \varepsilon_{r,J}) \in \mathbb{R}^J) \), and the part related to the velocities of the different layers \( (\mathbf{x}_v = (\beta_1, \ldots, \beta_K) \in \mathbb{R}^K) \).

As usual in inverse scattering problems [9], [24], a set of \( S \) electric line sources is considered in order to generate the illuminating fields. Their positions are here generically defined by referring to an integer parameter \( s \) varying in the set \( 1, \ldots, S \). All details will be provided in Section 5, where the different approaches here defined are applied to specific applications.

Another usual practice in inverse scattering problems [9] is related to the measure of the total or scattered field. It is very well known that axially moving cylinders can be treated as bianisotropic objects [2], [22] and that, in the presence of such scatterers, the axial components of the electric and magnetic fields are necessary and sufficient to determine all other components of the two fields [5], [20], [22]. For this reason, in this work, due to the possible motion of the scatterer, we assume that the axial components of the scattered electric and magnetic fields are measured by \( M \) sensors along a probing line. In this case, too, the positions of the sensors are defined by referring to an integer parameter \( m = 1, \ldots, M \). Again, specific details on the positions of the sensors will be provided in Section 5. The measures define the complex quantities \( E_{\text{meas}}^{z,\text{scatt}}(s,m,\mathbf{x}), H_{\text{meas}}^{z,\text{scatt}}(s,m,\mathbf{x}) \), for any incident field \( (s = 1, \ldots, S) \), for all measurement points \( (m = 1, \ldots, M) \). The measures of course depend on the unknown parameters \( \mathbf{x} \) of the inverse problem considered.

For any trial solution \( \mathbf{x}^t = (g_1^t, \ldots, g_T^t, \varepsilon_{r,1}^t, \ldots, \varepsilon_{r,J}^t, \beta_1^t, \ldots, \beta_K^t) \in \mathbb{R}^{I+J+K} \) of the inverse scattering problem, we assume to have at our disposal a forward scattering procedure \( (fsp) \) (which could be semianalytic, like the ones described in [22] and [5], or numeric) to determine the set of scattered components \( E_{z,\text{scatt}}^{fsp}(s,m,\mathbf{x}^t), H_{z,\text{scatt}}^{fsp}(s,m,\mathbf{x}^t) \). This can be done for any incident field \( (s = 1, \ldots, S) \) and for all measurement points \( (m = 1, \ldots, M) \). These quantities can be used to define a cost function indicating how far the trial solution \( \mathbf{x}^t \) is from the true solution \( \mathbf{x} \) of the inverse scattering problem [13]. In this way, in particular, the inverse scattering problem is recast as an optimization problem [13], [25].

The cost function is usually defined by exploiting all measures. For the case of interest it can be defined as follows:

\[
 f(\mathbf{x}, \mathbf{x}^t) = 
\]

\[
 = C_1(\mathbf{x}) \sum_{s=1}^{S} \sum_{m=1}^{M} |E_{z,\text{scatt}}^{fsp}(s,m,\mathbf{x}^t) - E_{z,\text{scatt}}^{\text{meas}}(s,m,\mathbf{x})|^2 + 
\]

\[
 + C_2(\mathbf{x}) \sum_{s=1}^{S} \sum_{m=1}^{M} |H_{z,\text{scatt}}^{fsp}(s,m,\mathbf{x}^t) - H_{z,\text{scatt}}^{\text{meas}}(s,m,\mathbf{x})|^2 , 
\]

\[
(29)
\]
where

\[ C_1(x) = \sum_{s=1}^{S} \sum_{m=1}^{M} |E_{zs,scatt}^{meas}(s, m, x)|^2, \]  

\[ C_2(x) = \sum_{s=1}^{S} \sum_{m=1}^{M} |H_{zs,scatt}^{meas}(s, m, x)|^2. \]

So far we have described the usual approach to the formulation of the optimization problem. When the problem of interest has to be solved without any restriction on the set of admissible values of \(\beta_i\) (apart from \(\beta_i \in (-1, 1)\), of course, and possibly \(\beta_i^2 \mu r, i \in (0, 1)\)) any simplified approach could be in trouble.

However, by taking account of the considerations reported in Sections 2 and 3 we can define another procedure which hopefully could provide better results, in a more efficient way, when the \(\beta_i\) values involved are known in advance to be sufficiently small in magnitude. A largest value of \(\beta_i\) equal to 0.01 allows us to conclude (see the last sentence of Section 3) that, independently of the media involved, the effects of the motion on the axial component of the electric field is negligible. Thus we can define a first-step optimization procedure aiming at the determination of \(x_g\) and \(x_d\) only, and that as a forward scattering procedure exploits a solver which assumes that all media are at rest (like the semianalytic one defined in [26] or a numerical method).

In this way the scattered components \(E_{zs,scatt}^{fsp,rest}(s, m, x_g^t, x_d^t)\) and \(H_{zs,scatt}^{fsp,rest}(s, m, x_g^t, x_d^t) = 0\) are determined and the cost function exploited in this first-step procedure is

\[ f_1(x, x_g^a, x_d^a) = C_1(x) \sum_{s=1}^{S} \sum_{m=1}^{M} |E_{zs,scatt}^{fsp,rest}(s, m, x_g^t, x_d^t) - E_{zs,scatt}^{meas}(s, m, x)|^2. \]

Once an approximate solution, \(x_g^a\) and \(x_d^a\) for the geometrical and dielectric parts, has been found we start a second step procedure whose target is to find an approximation of \(x_m\), and that as a forward scattering procedure exploits a solver able to manage problems involving axially moving media (like the semianalytic ones defined in [22] and [5] or a numerical method). In this way the scattered components \(E_{zs,scatt}^{fsp}(s, m, x_g^a, x_d^a, x_m^a)\) and \(H_{zs,scatt}^{fsp}(s, m, x_g^a, x_d^a, x_m^a) = 0\) are determined and the cost function exploited in this second step is

\[ f_2(x, x_g^a, x_d^a, x_m^a) = C_2(x) \sum_{s=1}^{S} \sum_{m=1}^{M} |H_{zs,scatt}^{fsp}(s, m, x_g^a, x_d^a, x_m^a) - H_{zs,scatt}^{meas}(s, m, x)|^2. \]

An approximate solution \(x_m^a\) is determined by minimizing \(f_2\). Overall, the approximate solution of the two-step inverse procedure is given by \(x_g^a, x_d^a, x_m^a\).

It could be interesting to notice that, when it is possible to use the two-step procedure, it could provide better results by requiring much less calculations than those required by the usual one-step approach. This is due, on the one hand, to the fact that the splitting of the unknowns could give significant advantages, since the complexity of inverse problems grows much more than linearly with the unknown number. On the other hand, the forward scattering procedure exploited in the first part of the two-step approach neglects any movement and is generally less complicated than the one adopted in the usual approach (for example the semianalytic procedure defined in [26] is much simpler than the one defined in [22]). As for the better results the two-step procedure could provide, we simply recall that in inverse scattering problems it is usually
of help to be able to identify the set of relevant data for the reconstruction at hand [9] and this is precisely what the two-step procedure does for inverse scattering problems of axially moving cylinders.

In this work, the cost functions previously defined for the different strategies are minimized by using an Artificial Bee Colony (ABC) algorithm [27], [28]. In such an approach (belonging to the family of swarm optimization methods), a population of trial solutions is iteratively modified according to stochastic rules until a convergence criterion is met. The population is initialized by randomly generating \( P \) trial solutions as

\[
x_{p,g}^{t(0)} = x_{\text{min},g} + U(0,1)(x_{\text{max},g} - x_{\text{min},g}),
\]

(34)

where the subscripts \( p = 1, \ldots, P \) and \( g = 1, \ldots, N_{\text{unk}} \) identify the \( g \)-th entry of the \( p \)-th trial solution. \( N_{\text{unk}} \) denotes the number of unknown parameters and in our inverse scattering problems we have \( N_{\text{unk}} = I + J + K \) when we use the general algorithm, \( N_{\text{unk}} = I + J \) in the first phase of the two-step approach and \( N_{\text{unk}} = K \) in its second phase. \( x_{\text{min},g} \) and \( x_{\text{max},g} \) are the lower and upper bounds on the \( g \)-th parameter, and \( U(0,1) \) is a function generating a uniformly distributed random number in the range \([0,1]\).

At the \( k \)-th iteration, the population is modified in three steps. In the first one (employed bee phase), \( P \) new trial solutions are generated by randomly changing all the elements of the current population. In particular, the \( g \)-th entry of the \( p \)-th trial solution is modified as

\[
v_{p,g} = x_{p,g}^{t(k)} + U(0,1)(x_{p,g}^{t(k)} - x_{h,g}^{t(k)}), \quad p = 1, \ldots, P,
\]

(35)

where the subscript \( h = 1, \ldots, P, h \neq p \), determines one of the \( P \) trial solutions. The indexes \( g \) and \( h \) are randomly chosen. The other components of the new trial solution \( v_p \) are the same as the ones of the corresponding original trial solution \( x_p^{(k)} \). A greedy selection is then applied, i.e., the new solution replaces the old one if it provides a lower value of the cost function. In the second step (onlooker bee phase), the obtained population is further modified by applying again equation (35) in order to obtain \( P \) new trial solutions. However, in this phase, the arrays to be changed are randomly selected on the basis of their fitness, i.e., they are chosen with a probability

\[
P_p = \frac{f_{\text{fit}}(x_p^{t(k)})}{\sum_{p=1}^{P} f_{\text{fit}}(x_p^{t(k)})},
\]

(36)

where the fitness function \( f_{\text{fit}} \) is related to the cost function \( f \) by

\[
f_{\text{fit}}(x_p^{t(k)}) = \frac{1}{f(x_p^{t(k)}) + 1}.
\]

(37)

In the cases we consider the cost function \( f \) is taken from (29), (32) or (33). A greedy selection is applied in this case, too, for propagating the best solutions to the next population. Finally, in the last step (scout bee phase) the algorithm check if the trial solutions have not been improved for a fixed number of iterations \( K_{\text{lim}} \). In this case, the stagnating solutions are abandoned and new arrays are randomly generated by using equation (34). The algorithm is stopped when a maximum number of iterations, \( K_{\text{max}} \), is reached or when the best solution, \( x_b^{t(k)}, b = 1, \ldots, P \), has not been significantly improved for a fixed number of iterations \( K_{\text{conv}} \), i.e., when the following condition is satisfied

\[
\frac{f(x_b^{t(k) - K_{\text{conv}}}) - f(x_b^{t(k)})}{f(x_b^{t(k)})} \leq f_{\text{conv}},
\]

(38)

being \( f_{\text{conv}} > 0 \) a fixed threshold.
Relative errors on $\varepsilon_r$, $\beta_1$

<table>
<thead>
<tr>
<th>General algorithm</th>
<th>0.011</th>
<th>0.051</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-step procedure</td>
<td>0.13</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Table 2: Relative errors (averaged on 10 runs) on the reconstruction of the unknown parameters. Single-layer cylinder.

5 Numerical reconstructions of multilayer elliptic cylinders by using different approaches

As pointed out in the Introduction, the reconstructions considered in this paper could be of interest in many applications, ranging from astrophysics to engineering. For this reason, we could consider different experimental configurations, with sources and sensors placed at very large distances from the moving scatterer or very close to it. In order to simplify the following analysis we consider an experimental configuration which is particularly useful in engineering applications. Fortunately, this choice does not limit in a significant way the range of velocities we can consider. As a matter of fact, in the open literature we can find plasma physics applications with sources and sensors close to the plasma columns moving at relativistic velocities [5] (see, in particular, Figure 10 for the experimental setup, and all other figures showing that $\beta$ could be as high as 0.5) and engineering applications of pneumatic pipes with similar experimental configurations but much smaller axial velocity values [8]. For this reason, in this section we analyse the performances of the two approaches defined in Section 4 for the reconstruction of the permittivity and velocity profiles of multilayer elliptic cylinders with sources and sensors in close proximity to the scatterers.

Our choice of the shape of the cylinder, on the one hand, is related to the fact that in this way we could exploit very efficient semianalytic forward scattering procedures [22], [26]. On the other hand, among the canonical scatterers allowing the definition of efficient procedures the multilayer elliptic cylinder is the most general one. As for the other parameters, we will consider values which are typically considered in microwave inverse scattering procedures [13], [14] in the presence of scatterers at rest since such procedures, so far, have not been applied to problems involving axially moving objects.

We begin with a very simple example, showing the obvious limitations of the two step approach when the axial velocity is significant. Let us consider a homogeneous elliptic scatterer characterized by $d = 0.15\lambda_0$, $a_1 = 0.3\lambda_0$, $\varepsilon_r,1 = 4$, $\beta_1 = 0.4$. The cylinder is illuminated by an electric line source whose features are the same as in Section 3, so that, for this simple case, we have $S = 1$. The axial field components are assumed to be measured by using $M = 36$ sensors uniformly distributed on a circle of radius $r_M = 0.6\lambda_0$. In order to simulate a more realistic environment, the input data are corrupted by a Gaussian noise with zero mean value and variance corresponding to a signal-to-noise ratio (SNR) of 20 dB.

In this simple example of reconstruction, we consider as unknowns just the two real parameters $\varepsilon_r,1$ and $\beta_1$ (the search ranges are $[1, 5]$ and $[0, 0.5]$, respectively). The parameters of the ABC algorithm have been set equal to $P = 3N_{unk}$, $K_{lim} = 0.5N_{unk}P$ (a minimum value of 5 is also used for this parameter), $K_{max} = 200$, $K_{conv} = 20$, $f_{conv} = 0.01$. Such values have been chosen on the basis of the suggestions available in the literature [29].

The results provided by the two considered reconstruction procedures are summarized in Table 2. In particular, the relative errors on the two parameters, averaged on ten runs, are reported for both the general and two-step approaches. As can be seen, the general procedure is able to correctly reconstruct the two unknowns, whereas the two-step procedure provides high
Figure 4: Behaviour of the real and imaginary parts of $E_z$ along the circle where the sensors are placed. Two plots refer to the exact solution while the others are computed by assuming $\beta_1 = 0$. There is a very significant difference between the two complex solutions.

errors.

The essential reason for the failure of the two-step approach is that it uses a wrong algorithm as a forward scattering procedure, in which all movements are neglected. In this case, even with the exact value of $\varepsilon_{r,1}$, the differences between $E_{z,scatt}^{fsp,rest}(s=1,m,x_t=x_g, x_d=x_d)$ and $E_{z,scatt}^{meas}(s=1,m,x)$ are very relevant, as it is clearly shown in Figure 4. It is worth noting that in this simple case both algorithms converge in all the considered runs.

Now we consider some more challenging examples. In all cases considered in the following we assume that the largest value of $\beta_i$, $i = 1, \ldots, N$, is below the limit of 0.01 indicated in Section 3. The imaging configuration is similar to that of the previous example. In particular, a multi-view arrangement where the incident field is generated by $S = 4$ electric line sources uniformly distributed on a circumference of radius $r_s = 0.8\lambda_0$ is considered. The axial components are measured in $M = 36$ points uniformly distributed on a circle of radius $r_M = 0.6\lambda_0$. The scattered field data are numerically computed and corrupted with a zero-mean Gaussian noise with $SNR = 20$ dB.

In the first case, a two-layer cylinder, characterized by $d = 0.15\lambda_0$, $a_1 = 0.25\lambda_0$, $a_2 = 0.4\lambda_0$, $\varepsilon_{r,1} = 4$, $\varepsilon_{r,2} = 2$, and $\beta_2 = 0$, is considered. The velocity of the inner layer, $\beta_1$, has been varied between 0.0001 and 0.01. Initially, only the dielectric properties of the two layers and the velocity of the inner core have been reconstructed by using the two procedures detailed in the previous Section. The bounds on the unknowns are similar to those used in the first example, except that the maximum velocity for $\beta_1$ is set to 0.1.

The reconstruction results are provided in Table 3, which reports the mean relative errors on the reconstruction of the unknowns, the mean number of iterations needed to reach the convergence, the mean number of cost function evaluations, and the mean value of the final cost function of the best individual.

As can be seen, in all cases the two-step approach outperforms the general procedure in
Table 3: Relative errors on the reconstruction of the unknown parameters and computational
data (averaged on 10 runs). Two-layer cylinder (dielectric and velocity parameters only).

<table>
<thead>
<tr>
<th>$\beta_1 = 10^{-2}$</th>
<th>Mean relative errors</th>
<th>Mean number of cost function evaluations</th>
<th>Mean cost function value of best ind.</th>
</tr>
</thead>
<tbody>
<tr>
<td>General algorithm</td>
<td>$\varepsilon_{r,1}$</td>
<td>$0.040$</td>
<td>$68.4$</td>
</tr>
<tr>
<td></td>
<td>$\beta_1$</td>
<td>$0.048$</td>
<td>$1247.1$</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_{r,2}$</td>
<td>$0.024$</td>
<td>$0.11$</td>
</tr>
<tr>
<td>Two-step procedure</td>
<td>$0.0061$</td>
<td>$0.0060$</td>
<td>$41$ (step 1)</td>
</tr>
<tr>
<td></td>
<td>$0.0053$</td>
<td>$27.6$ (step 2)</td>
<td>$170.8$ (step 2)</td>
</tr>
<tr>
<td></td>
<td>$0.0150$</td>
<td>$0.0065$</td>
<td>$509.7$ (step 1)</td>
</tr>
<tr>
<td></td>
<td>$0.0150$</td>
<td>$0.0065$</td>
<td>$0.11$ (step 1)</td>
</tr>
<tr>
<td></td>
<td>$0.0150$</td>
<td>$0.0065$</td>
<td>$0.11$ (step 2)</td>
</tr>
<tr>
<td>$\beta_1 = 10^{-3}$</td>
<td>General algorithm</td>
<td>$0.059$</td>
<td>$77.2$</td>
</tr>
<tr>
<td></td>
<td>$0.072$</td>
<td>$1414.4$</td>
<td>$0.13$</td>
</tr>
<tr>
<td>Two-step procedure</td>
<td>$0.0076$</td>
<td>$0.0050$</td>
<td>$39.4$ (step 1)</td>
</tr>
<tr>
<td></td>
<td>$0.0065$</td>
<td>$38.7$ (step 2)</td>
<td>$238.1$ (step 2)</td>
</tr>
<tr>
<td></td>
<td>$0.0076$</td>
<td>$0.0065$</td>
<td>$490.3$ (step 1)</td>
</tr>
<tr>
<td></td>
<td>$0.0076$</td>
<td>$0.0065$</td>
<td>$0.11$ (step 1)</td>
</tr>
<tr>
<td></td>
<td>$0.0076$</td>
<td>$0.0065$</td>
<td>$0.098$ (step 2)</td>
</tr>
<tr>
<td>$\beta_1 = 10^{-4}$</td>
<td>General algorithm</td>
<td>$0.037$</td>
<td>$85.6$</td>
</tr>
<tr>
<td></td>
<td>$0.31$</td>
<td>$1506.1$</td>
<td>$0.29$</td>
</tr>
<tr>
<td>Two-step procedure</td>
<td>$0.0084$</td>
<td>$0.0072$</td>
<td>$44.8$ (step 1)</td>
</tr>
<tr>
<td></td>
<td>$0.0072$</td>
<td>$34.5$ (step 2)</td>
<td>$213.5$ (step 2)</td>
</tr>
<tr>
<td></td>
<td>$0.0084$</td>
<td>$0.0072$</td>
<td>$552.7$ (step 1)</td>
</tr>
<tr>
<td></td>
<td>$0.0084$</td>
<td>$0.0072$</td>
<td>$0.11$ (step 1)</td>
</tr>
<tr>
<td></td>
<td>$0.0084$</td>
<td>$0.0072$</td>
<td>$0.10$ (step 2)</td>
</tr>
</tbody>
</table>

In terms of accuracy in the reconstruction of the searched quantities. In particular, the results obtained by the general procedure for lower velocities are not satisfactory while those obtained by the two-step procedure are good. In fact, during the iterative minimization, the complete procedure tries to fit the values of both the electric and magnetic fields in the measurement points, which represent an oversized set of input data. However, since these data depend on the current solution in different ways (as clearly described in Section 2), it is obviously expected that significant numerical errors are present in the inverse solution due to the weak dependence of the electric field data from the target axial velocities (for low values of $\beta$). Moreover, the two-step procedure require less cost function evaluations and each of these often requires much less calculations than that adopted in the usual procedure. Thus, a faster reconstruction is obtained.

These considerations related to the accuracy of the reconstruction and to the computational effort required apply to all other cases that follow and, for this reason, will not be reported anymore. For completeness, Figures 5 and 6 provide, respectively, the behaviors of the cost functions and of the mean relative reconstruction errors versus the iteration number for the ten considered runs and for the two strategies in the case $\beta_1 = 0.001$.

The same configuration has been also considered for testing the capabilities of the algorithms in jointly reconstructing the constitutive and the geometrical parameters together with the velocity of the inner layer. $\beta_1$ is set to 0.001 in this case. For the general procedure, two cases are considered: in the first one, no a priori information is used, i.e., $\beta_1$ is searched in the range $[0, 0.5]$, while in the second one the velocity is limited to $\beta_1 \leq 0.1$. The obtained result are summarized in Table 4. As can be seen, the two-step procedure is able to reconstruct the velocity and the dielectric/geometrical features of the target with quite good accuracy, whereas the general algorithm (with the a-priori information about the range of the velocity) provides higher reconstruction errors. Moreover, the general strategy without a-priori information fails in obtaining accurate results.

Finally, a more complex target, composed by three layers, has been considered in the last example. The parameters of the target are the following: $d = 0.15\lambda_0$, $a_1 = 0.25\lambda_0$, $a_2 = 0.35\lambda_0$, $a_3 = 0.45\lambda_0$, $\varepsilon_{r,1} = 4$, $\varepsilon_{r,2} = 2$, $\varepsilon_{r,3} = 3$, $\beta_1 = 0.001$, $\beta_2 = 0.005$, and $\beta_3 = 0$. Similarly to the other cases, the data have been numerically simulated and corrupted with a Gaussian noise ($SNR = 20$ dB). The measurement configuration is the same as in the previous example. In this
Figure 5: Behavior of (a) the cost function and of (b) the mean relative reconstruction error versus the iteration number for the considered runs. General strategy. Two-layer cylinder. \( \beta_1 = 0.001 \).

Figure 6: Behavior of the cost function ((a) first step and (b) second step) and of the mean relative reconstruction errors ((c) first step and (d) second step) versus the iteration number for the considered runs. Two-step strategy. Two-layer cylinder. \( \beta_1 = 0.001 \).
Table 4: Relative errors on the reconstruction of the unknown parameters and computational data (averaged on 10 runs). Two-layer cylinder (dielectric, geometrical and velocity parameters).

<table>
<thead>
<tr>
<th>Mean relative errors</th>
<th>Mean number of iterations</th>
<th>Mean number of cost function evaluations</th>
<th>Mean cost function value of best ind.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{r,1}$</td>
<td>$\beta_1$</td>
<td>$\varepsilon_{r,2}$</td>
<td>$\beta_2$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.14</td>
<td>24.08</td>
<td>0.14</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.16</td>
<td>0.43</td>
<td>1960.7</td>
</tr>
<tr>
<td>General algorithm (no a-priori information)</td>
<td>64.8</td>
<td>0.37</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean relative errors</th>
<th>Mean number of iterations</th>
<th>Mean number of cost function evaluations</th>
<th>Mean cost function value of best ind.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{r,1}$</td>
<td>$\beta_1$</td>
<td>$\varepsilon_{r,2}$</td>
<td>$\beta_2$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.12</td>
<td>0.22</td>
<td>0.13</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.098</td>
<td>0.36</td>
<td>2286</td>
</tr>
<tr>
<td>General algorithm (with a-priori information)</td>
<td>75.6</td>
<td>0.21</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean relative errors</th>
<th>Mean number of iterations</th>
<th>Mean number of cost function evaluations</th>
<th>Mean cost function value of best ind.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{r,1}$</td>
<td>$\beta_1$</td>
<td>$\varepsilon_{r,2}$</td>
<td>$\beta_2$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.036</td>
<td>0.027</td>
<td>0.031</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.017</td>
<td>0.076</td>
<td>61.2 (step 1)</td>
</tr>
<tr>
<td>Two-step procedure</td>
<td>34.4 (step 2)</td>
<td>1485.3 (step 1)</td>
<td>0.11 (step 1)</td>
</tr>
</tbody>
</table>

Table 5: Relative errors on the reconstruction of the unknown parameters and computational data (averaged on 10 runs). Three-layer cylinder (dielectric and velocity parameters only).

<table>
<thead>
<tr>
<th>Mean relative errors</th>
<th>Mean number of iterations</th>
<th>Mean number of cost function evaluations</th>
<th>Mean cost function value of best ind.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{r,1}$</td>
<td>$\beta_1$</td>
<td>$\varepsilon_{r,2}$</td>
<td>$\beta_2$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.037</td>
<td>0.28</td>
<td>0.99</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.22</td>
<td>0.22</td>
<td>86.2</td>
</tr>
<tr>
<td>General algorithm</td>
<td>0.063</td>
<td>2604.2</td>
<td>0.18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean relative errors</th>
<th>Mean number of iterations</th>
<th>Mean number of cost function evaluations</th>
<th>Mean cost function value of best ind.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{r,1}$</td>
<td>$\beta_1$</td>
<td>$\varepsilon_{r,2}$</td>
<td>$\beta_2$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.0018</td>
<td>0.029</td>
<td>0.057</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.027</td>
<td>0.057</td>
<td>50.3 (step 1)</td>
</tr>
<tr>
<td>Two-step procedure</td>
<td>47.2 (step 2)</td>
<td>922.3 (step 1)</td>
<td>0.10 (step 1)</td>
</tr>
</tbody>
</table>

case, the considered unknowns are the dielectric properties of the three layers and the velocities of the two inner media. In the reconstruction processes both the unknown velocities are limited to the range $[0, 0.1]$.

The results provided by the two algorithms are reported in Table 5. As can be seen, the proposed two-step procedure is able to provide good reconstructions of the dielectric properties and of the layer velocities. For completeness, an example of the evolution of the best individual of the population for a run of the two-step strategy is reported in Figure 7 (a) and (b) concerning, respectively, the first and the second step of the procedure.

6 Conclusions

Starting from an analysis on the different effects of the axial movement on the co-polarized and cross-polarized scattered waves, two inverse scattering procedures for the reconstruction of the permittivity and of the velocity profiles are defined. One of these is classical and manages all the unknowns in the same way by defining a global optimization problem. The second procedure, on the contrary, taking account of the indicated different effects of the movement, splits the problem in two parts. In the first one the permittivity profile is determined by neglecting all axial movements and provides very good results, in a very efficient way, for an impressive range of axial speed values, range which includes most of the speed values of interest in practical applications. In the second part, just the velocity profile is determined. For relativistic movements the two-step procedure is in trouble and it is shown that the classical one can provide satisfactory results.
“Electromagnetic inverse scattering of axially moving cylindrical...”

Figure 7: Example of evolution of the best individual of the population for a run of the two-step strategy. (a) First step (reconstruction of the dielectric properties) and (b) second step (reconstruction of the velocities). Three-layer cylinder.

References


